Characterizing the Neurocognitive Mechanisms of Arithmetic

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Characterizing the Neurocognitive Mechanisms of Arithmetic

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Dissertation submitted for the degree of
Doctor of Philosophy

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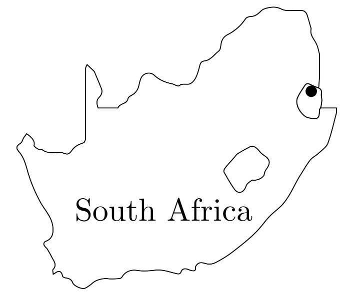
Introduction

- **Mathematics**: a remarkable human invention.
- The fundamental language of science.

- Most elementary branch: **Arithmetic** – invented 50,000 years ago.

Tally sticks: store and transmit numerical information.



Central Market – Belo Horizonte

NeuroSpin

How are two numbers combined into a third?

$3+5=8$

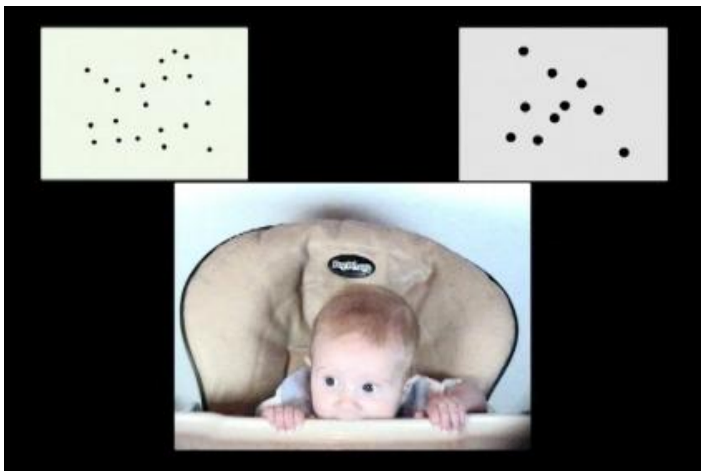
- Sophisticated abstract concepts & complex network of brain regions.
- **My goal: characterize the neurocognitive mechanisms of arithmetic.**
- **Time-resolved**: combining methods that allow for high temporal resolution of behavior/brain activity and high anatomical precision.
- **Decompose the fast dynamics of the underlying processing stages**.
    - Encoding of the operands, calculation *per se* and response.

- **General overview of the state of the art: progress and limitations.**

Foundations of arithmetic thinking

- **The Number Sense** *(Dantzig, 1967; Dehaene, 1999)*
  - Similar to the intuitions we have for time and space.
  - Spatially organized representation: *mental number line* *(Galton, 1881; Dehaene et al. 1993).*

Number discrimination in babies
(Izard et al., 2009)



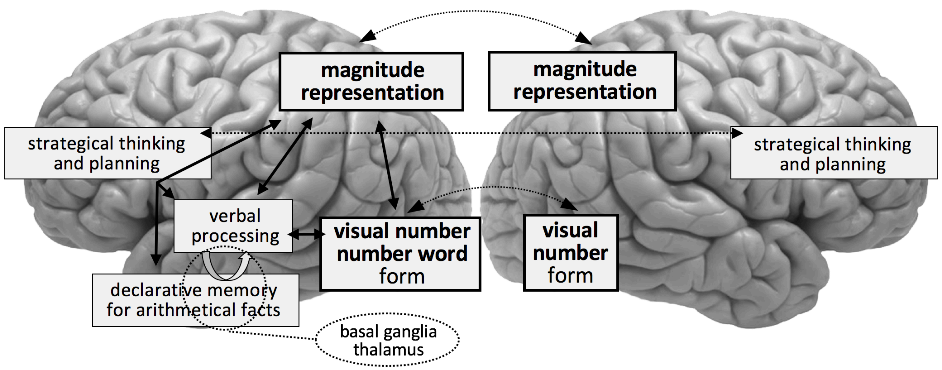
Democratic decisions in baboons
(Piantadosi & Cantlon, 2017)

- **Evolutionary origin**: protect cubs, select preys, mating, etc.
- **Biological origin**: neurons tuned to numerosity in monkeys *(Nieder & Miller, 2004).*
  Number network in monkeys and humans largely overlap in parietal and frontal areas.

Brain networks for arithmetic processing

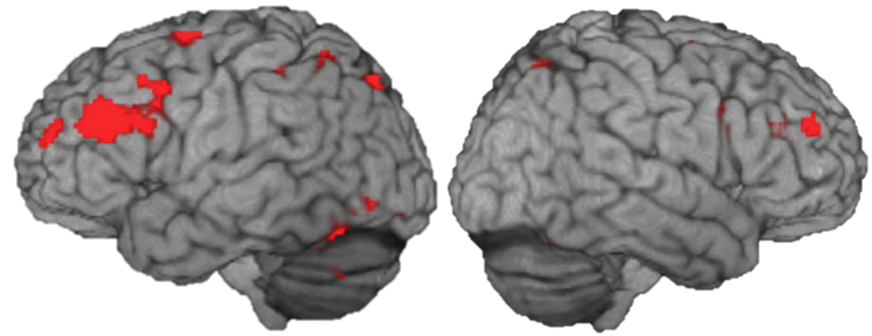
Putative network - neuropsychology

(Dehaene & Cohen, 1995)



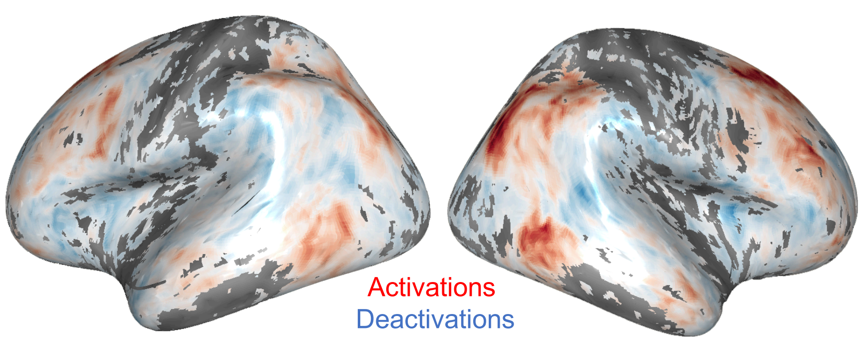
fMRI meta-analysis

(Arsalidou & Taylor, 2011)



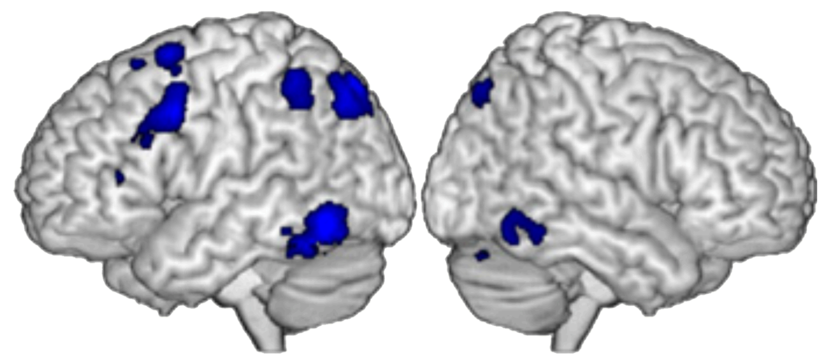
fMRI activations for number words in natural speech

(Huth et al.,2016)



Similar network engaged in high-level mathematics

(Amalric & Dehaene, 2016)

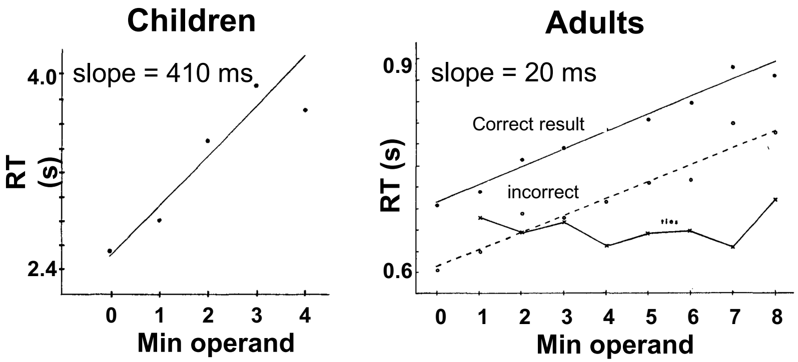


- Precise roles of each region and temporal dynamics still largely unknown.

How does it work? Cognitive models of arithmetic:

- **Problem-size effect**: RTs increase as a function the size of the operands.

- **The '*min*' counting model**
  (Groen & Parkman, 1972).
  - ***Min* operand best predictor of RT**



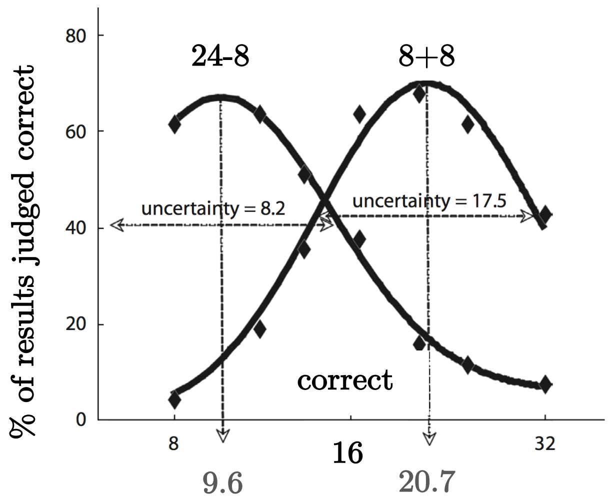
- **Fact retrieval models** *(Ashcraft, 1992, etc.)*
  - Commutativity - half of the table is memorized: L+S *(Butterworth et al., 2001).*

- **Compact counting procedures**
  - Scrolling an ordered representation, such as the ***mental number line***.
    (Barrouillet & Thevenot, 2013)

Calculation as *’movement’* along the mental number line

Operational Momentum effect

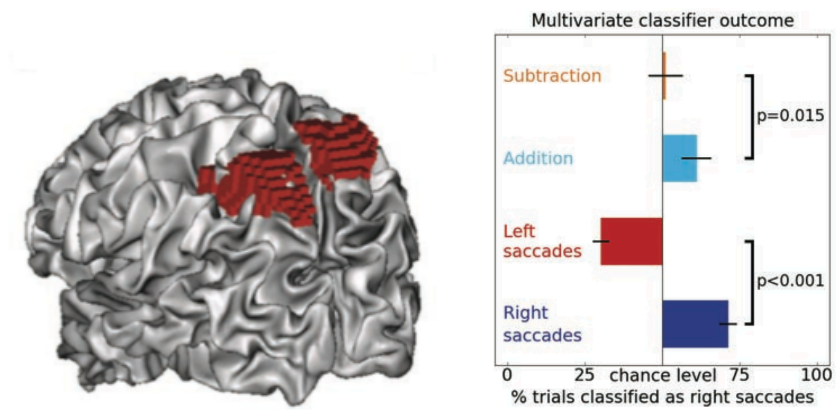
(McCrink et al., 2007)



- **Additions:** overestimated
- **Subtractions:** underestimated

Decoding left vs. right saccades generalizes to sub vs. add in pSPL

(Knops et al, 2009)



VIP-LIP vector addition for eye movement control
(Pouget, et al., 2002)

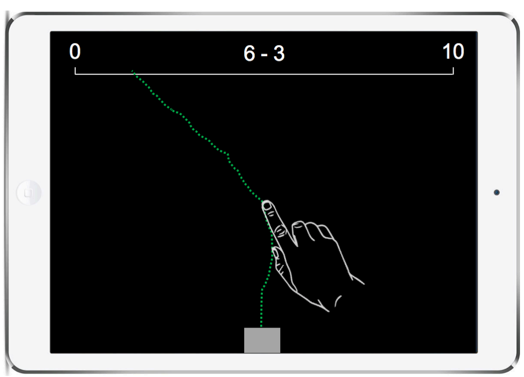
- Specific mechanism and temporal dynamics remain elusive.
  During calculation or post-calculation?

Experimental contributions

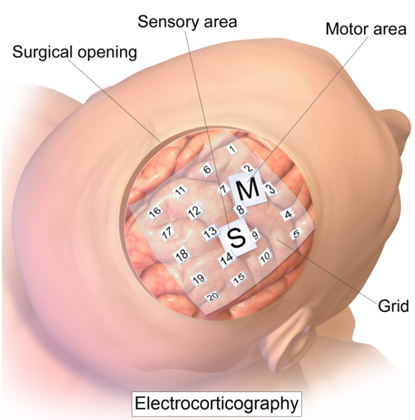
- **Progress has been methodologically impeded by relying on:**
  Summary measures of the entire processing chain.
  - **Mental chronometry:** *blind to the absolute timing and order of the stages.*
  - **fMRI**: *low temporal resolution & relatively coarse functional specificity.*

- **My approach, developed in 3 main studies:**
  1. Dissect the covert stages of simple arithmetic.
  2. Re-evaluate the neural correlates of mental calculation.
  3. Decode the brain processes and underlying representational codes.

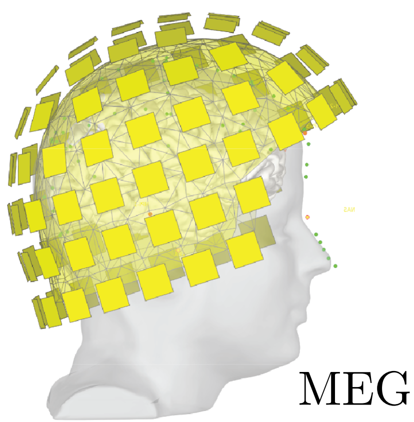
1. Finger-tracking



2. ECoG



3. Time-resolved MVPA

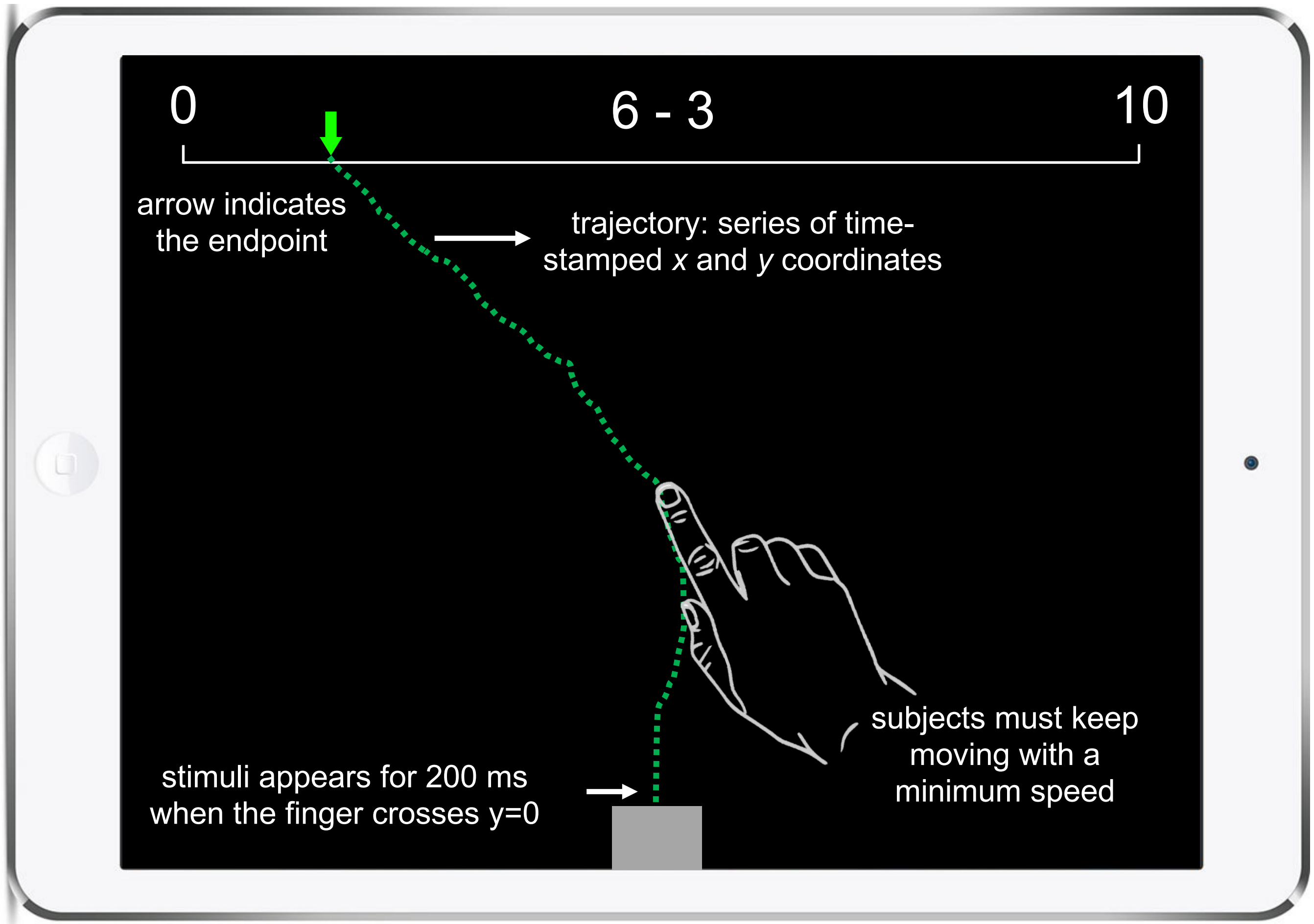


Chapter 2. **Finger-tracking reveals the covert stages of mental arithmetic**

Dror Dotan
Lecturer & **finger-tracker**
Tel Aviv University

Experimental Setup

30 adults, students, right-handed



Assumption

finger trajectories track the ongoing decision process underlying a cognitive task

Chapter 2. **Finger-tracking reveals the covert stages of mental arithmetic**

- Are the two operands processed serially or in parallel?
- Is there a stage whose duration increases linearly with the size of the numerical quantities, as implied by counting models?
- Can we determine the moment when the visuospatial biases underlying addition and subtraction occur?

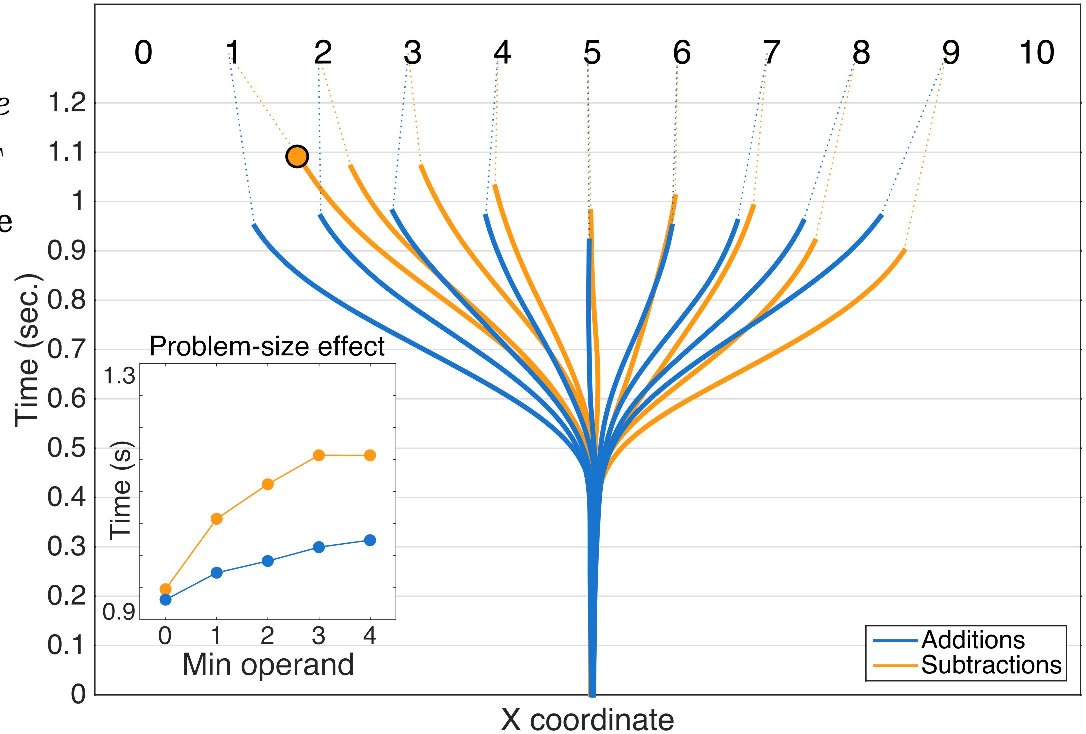
Pinheiro-Chagas, Dotan, Piazza & Dehaene (2017) Open Mind.

Averaged trajectories by result

movement time

equivalent to RT

summary measure

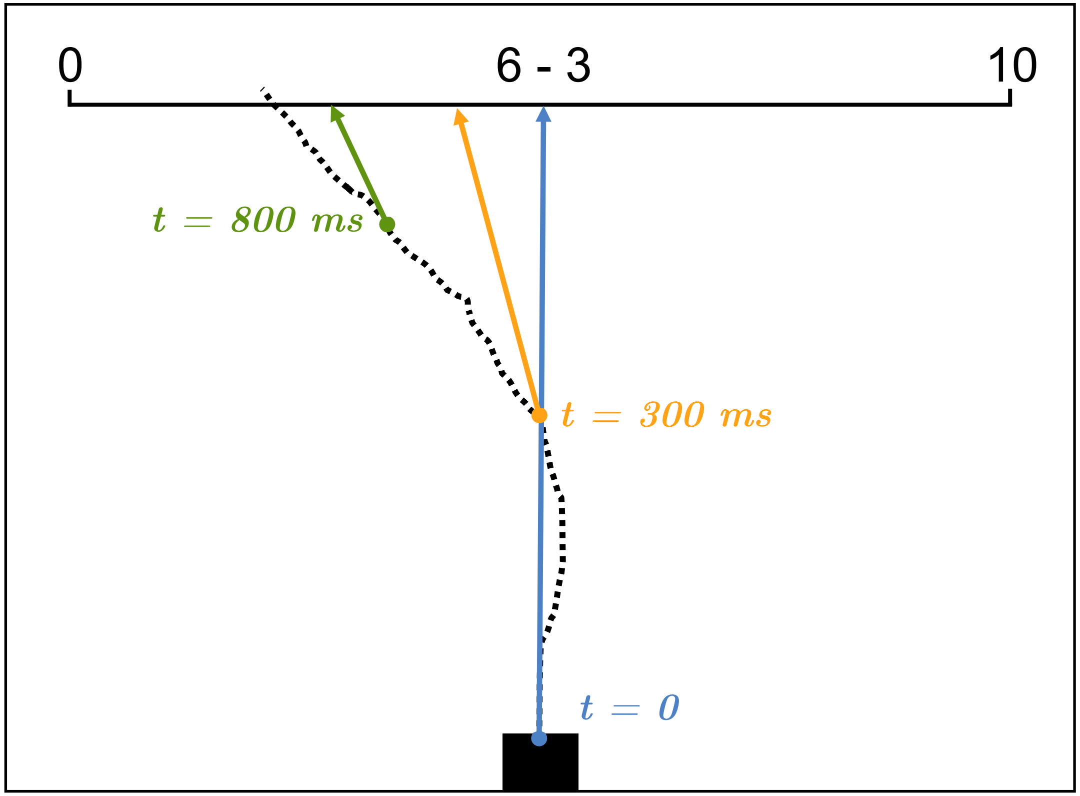


- The ***min*** operand is the best predictor of MT in additions & subtractions slope: 55 ms *sub*; 25 ms *add; Replicates Groen & Parkman (1972), etc.*

Research in the past 40 years would stop at this stage to formulate cognitive models of arithmetic – problem-size effect

Decomposing the calculation task

Time-resolved multiple regression

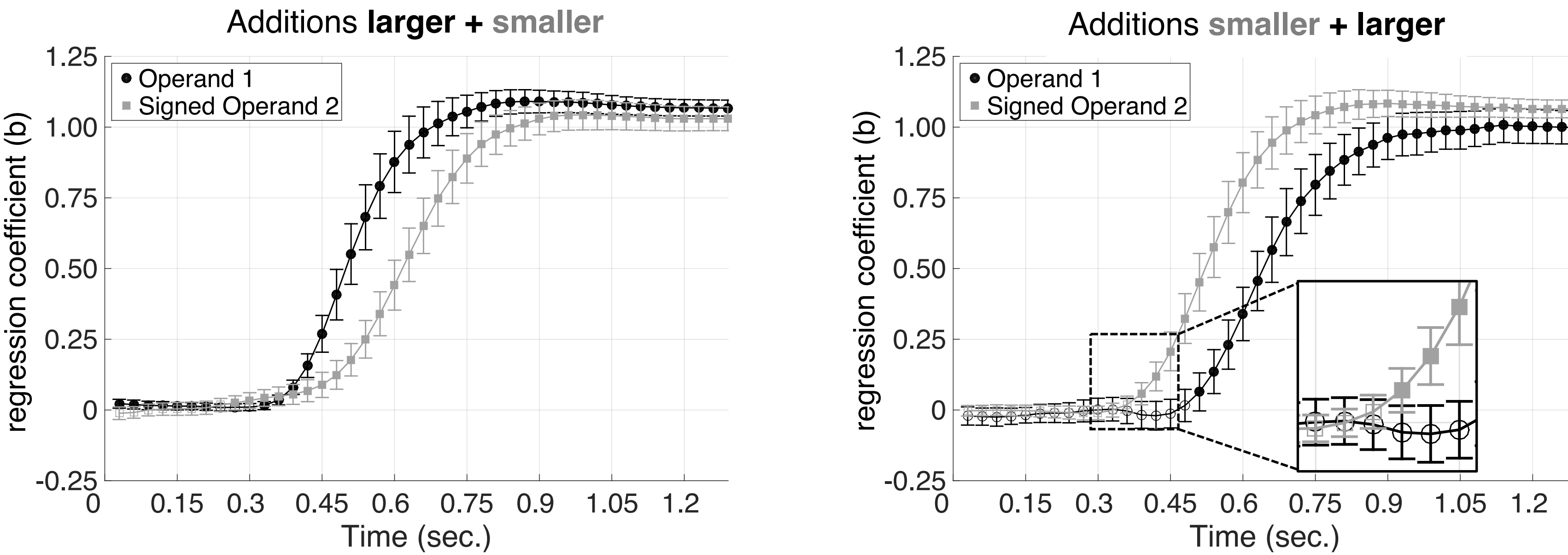


Implied endpoints: where the subject is aiming at each time t

- **Multiple Regression** at each time sample *(30 ms), per subject.*
  - ***Dependent variable****: implied endpoint.*
  - ***Predictors****: operand 1, operand 2, operation, etc.*

Are the operands processed serially or in parallel?

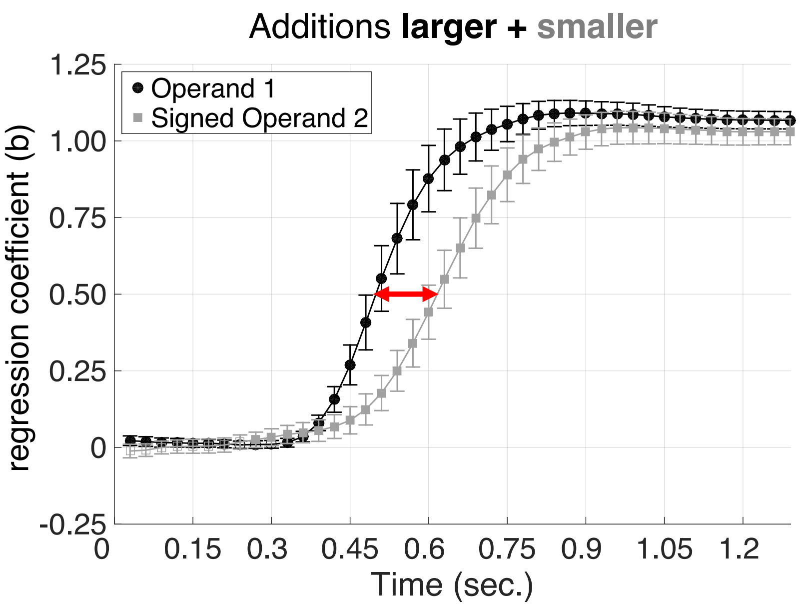
Serial processing of the operands in additions

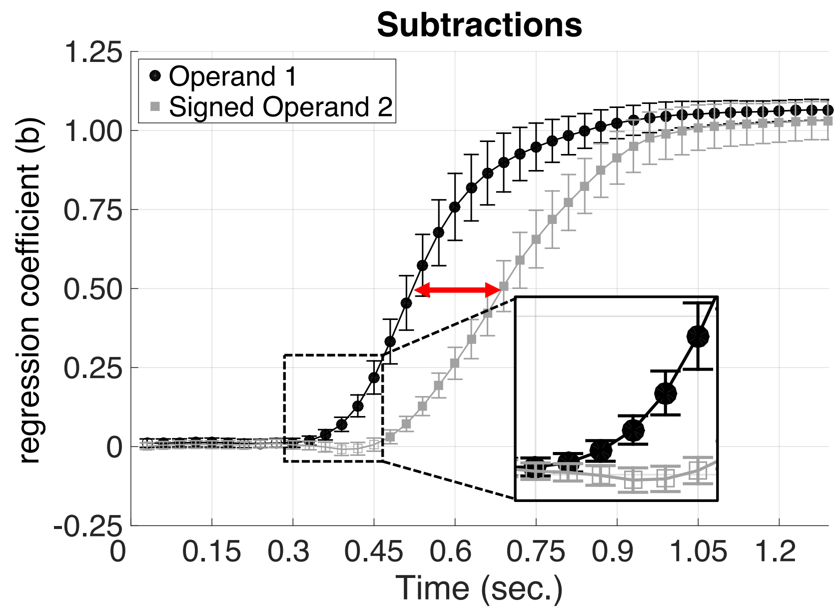


- **Reordering of the operands**: subjects first point to the larger operand irrespectively of the its location. *The effect is present in all 30 subjects.*

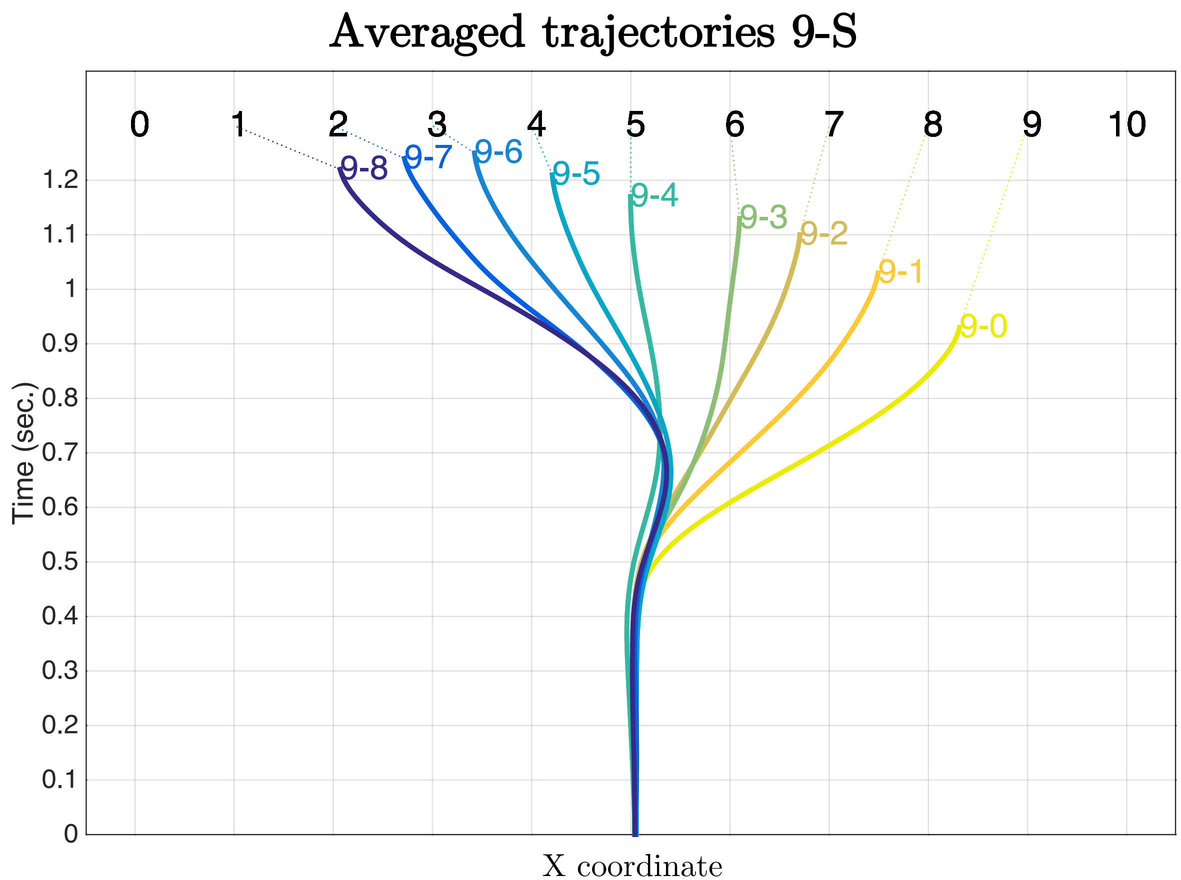
- Cost in MT of 14 ms on average. *Replicates (Butterworth et al., 2001).*

Higher serial processing of the operands in subtractions





Direct visualization of the serial processing of the operands

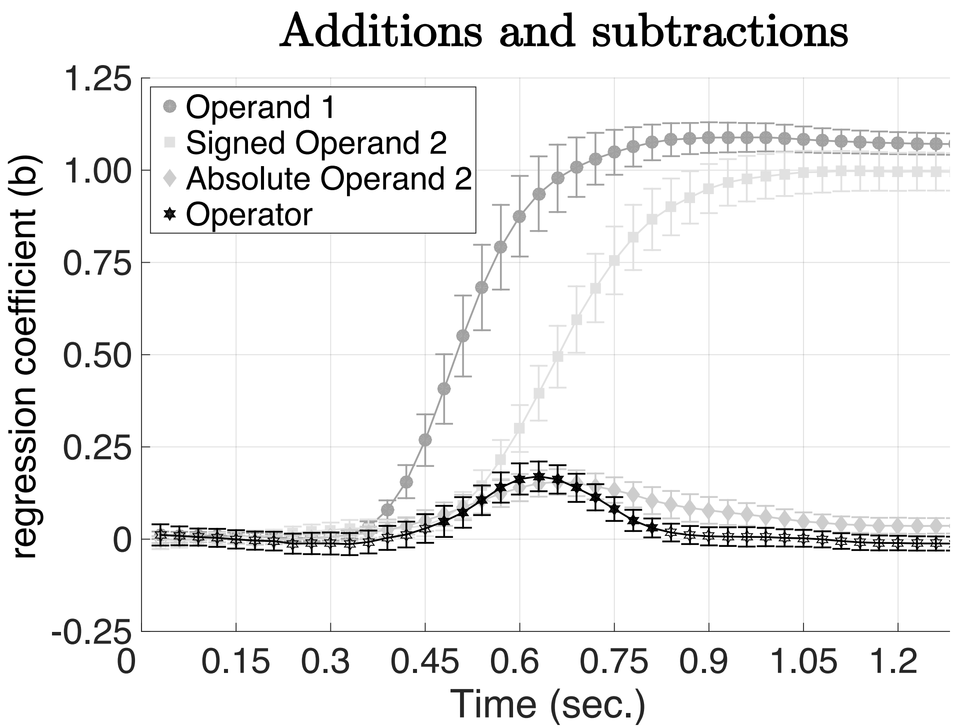


- Subjects first point to the larger operand, then deviate to the correct result.
- Deviation unfolds **serially** and **proportionally** to the size of the ***min*** operand.
  In both subtractions and additions (5-S, 5+S, etc.)

Passing through intermediate stages?

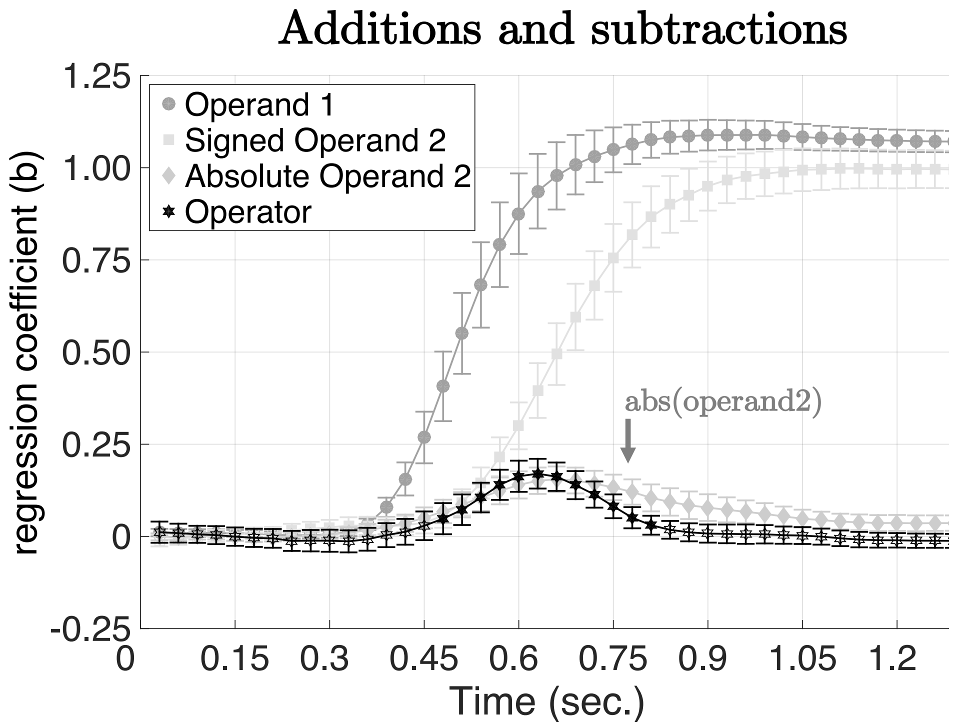
What is the time-course of the visuospatial biases in addition and subtraction?

Transient OM effect at the time of processing the *min* operand



- $+$ and $-$ signs distorts the finger to the right & left sides (larger & smaller numbers).
- ***During calculation***, *not post-calculation.*

Transient activation of the absolute value of the subtrahend



- **Additional stage in subtraction**: discard the absolute value of the *min*.

Chapter 2. **Discussion and conclusions**

- The **operands** are **processed serially**: larger operand first independent of its location: **direct visualization** of the **reordering effect** for additions S+L.

- The deviation from the larger operand to the correct result unfolds serially and **proportionally** to the **size** of the ***min*** **operand** (intermediate stages?).

- **Transient OM effect** at the time of the **integration of the *min***: visuospatial attention system recruited during the calculation.

- **Support for a model in which single-digit additions and subtractions are computed by a stepwise displacement on the mental number line.**
  Compatible with compact counting procedures & retrieval by tabular search.

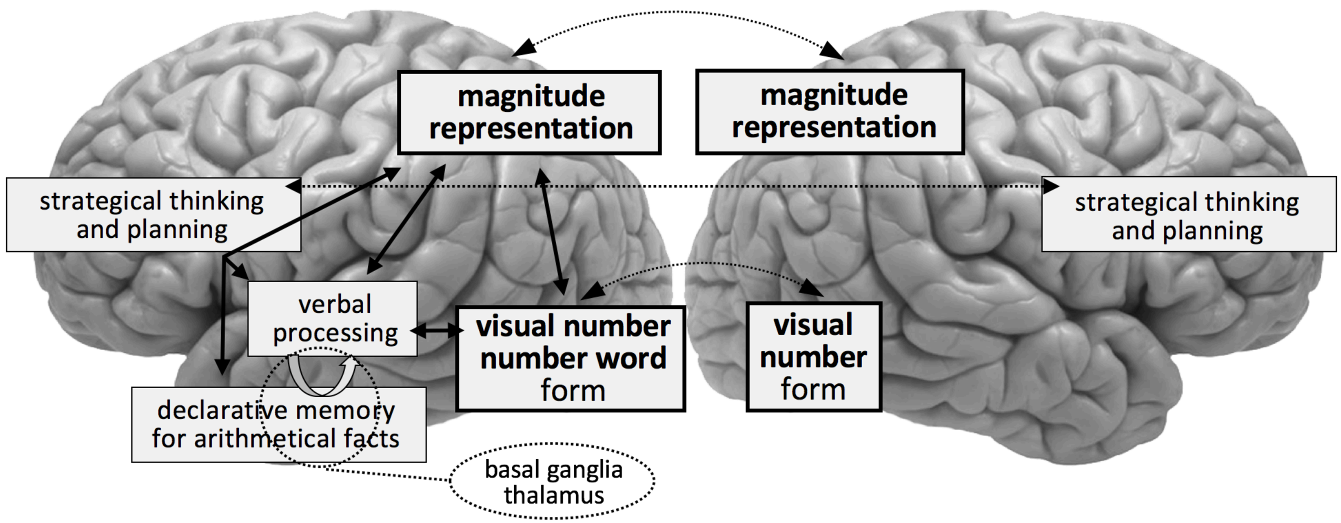
Pinheiro-Chagas, Dotan, Piazza & Dehaene (2017) Open Mind.

What are the neural correlates?

Traditional view

Putative network - neuropsychology

(Dehaene & Cohen, 1995)

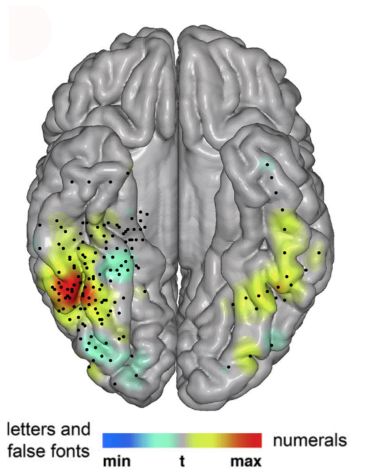


- **Lateral parietal cortex**: main hubs for magnitude processing and calculation. BOLD activity in the **IPS** and **SPL increase** as a function of problem-size. *(Dehaene et al., 1999; Kanjlia et al., 2016, etc.)*
- **Ventral temporal cortex:** visual recognition of numerals …

Arithmetic processing in the *ventral* stream

'Number Form Area' in the pITG

(Shum et al. 2013)

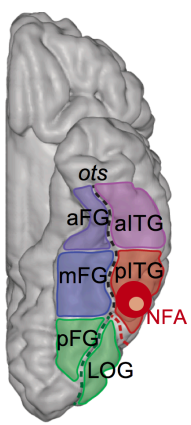


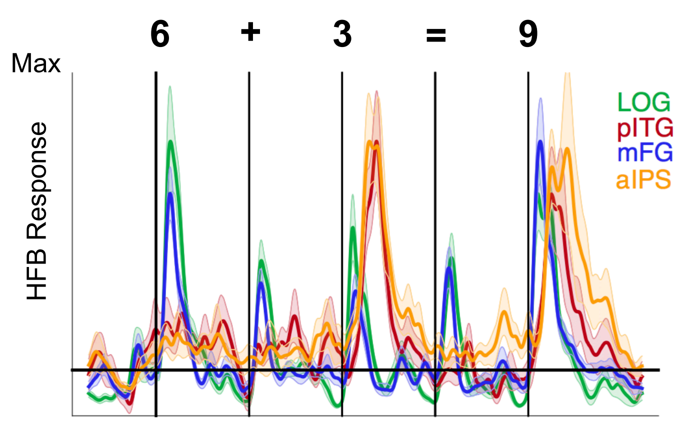
Two distinct neuronal populations in the pITG

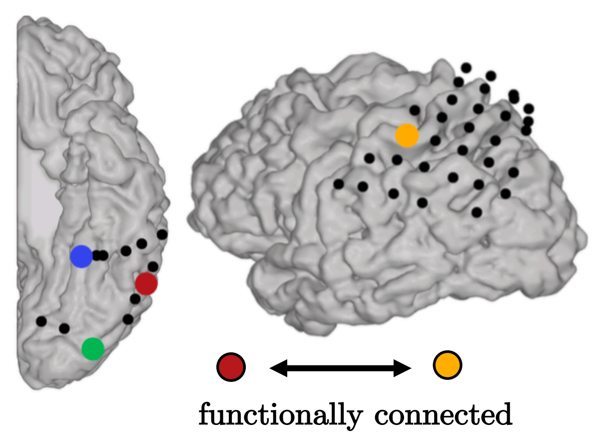
(Daitch et al., 2016)

NFA: response to numerals is *context independent*

Adjacent population – $\mathbf{pITG_{math}}$: response to numerals depends on *calculation*







Chapter 3. **Brain mechanisms of arithmetic: a crucial role for ventral temporal cortex**

Josef Parvizi

Amy Daitch

Stanford University

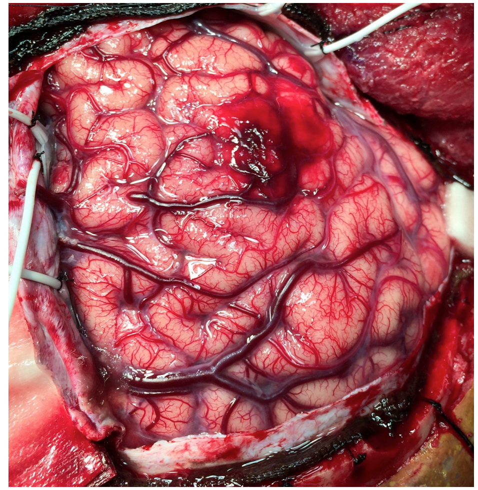
Chapter 3. **Brain mechanisms of arithmetic: a crucial role for ventral temporal cortex**

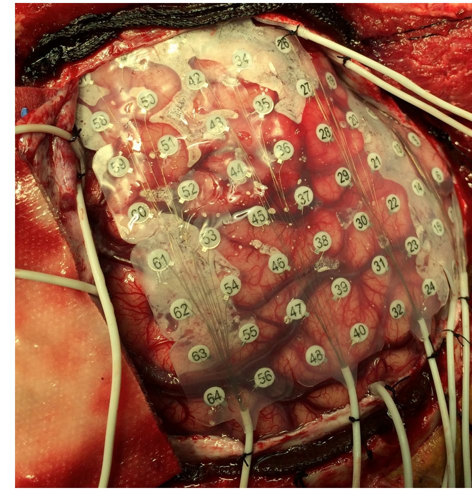
- Test **if**, **how** and **when** numerical features ***(problem-size)*** modulate the activity in calculation-selective neuronal populations in the **LPC** and **VTC**.

- Predictions:
    - **IPS:** parametrically increase of the activity with problem-size.
    - **pITG: ?**
        - If digit recognition only: early burst, constant for all problems.
        - If top-down attentional modulation– increase with problem-size.
        - Unpredicted role in calculation?

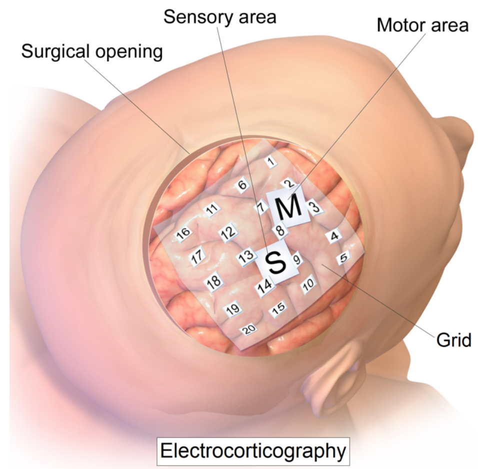
Pinheiro-Chagas, Daitch*, Parvizi & Dehaene (under review)*

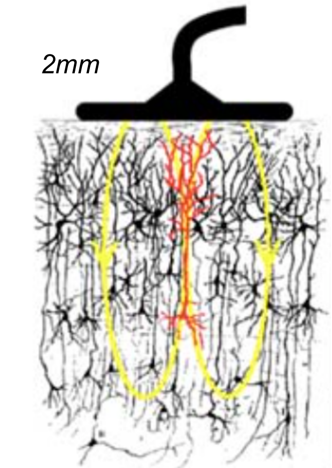
Electrocorticography (ECoG)



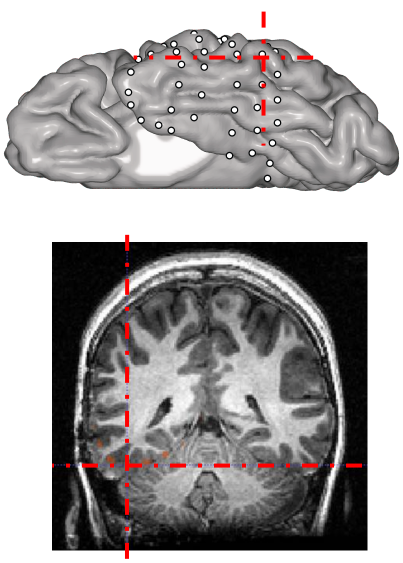


Electrocorticography (ECoG)

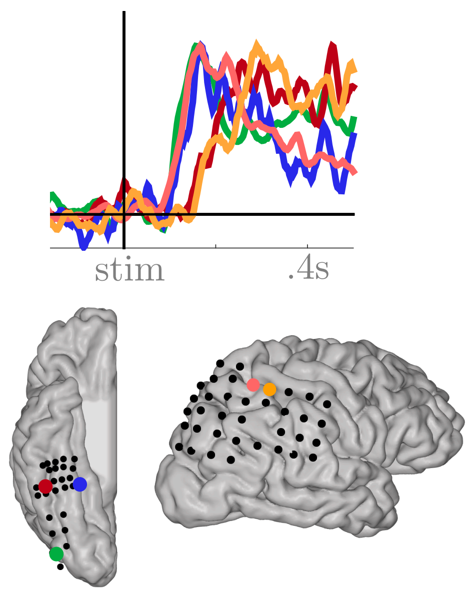




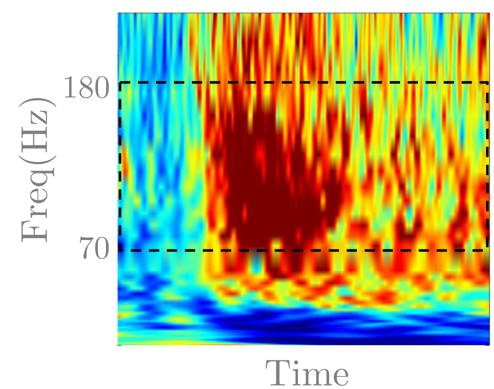
High spatial precision



High temporal resolution



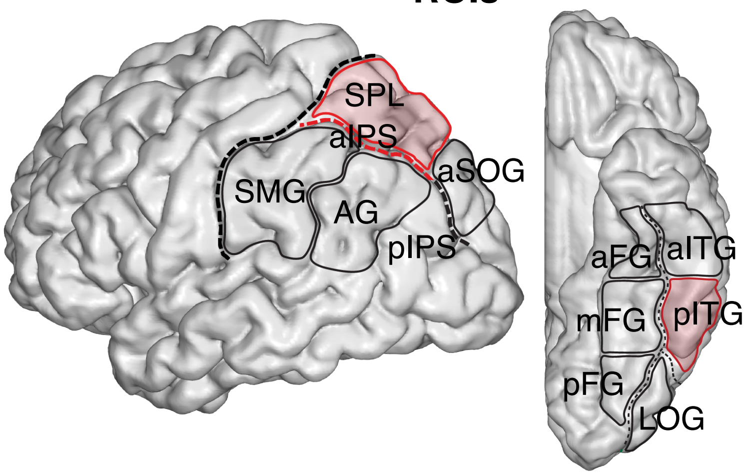
Rich frequency content



Task, ROIs, selectivity and problem-size effect

10 subjects with coverage in VTC and LPC

ROIs



Math

13+5=17

80 trials
Min op: 1:9
Max op: 10:87

Memory

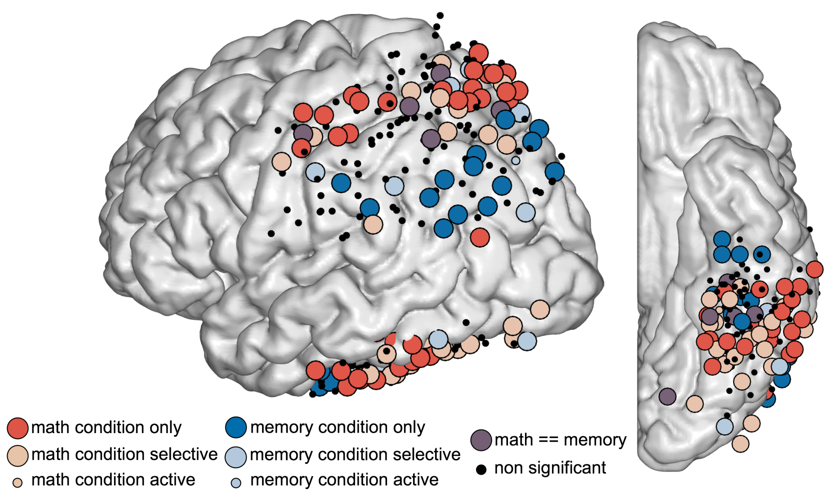
I ate fruit today

control
sentence comprehension
50 trials

RT

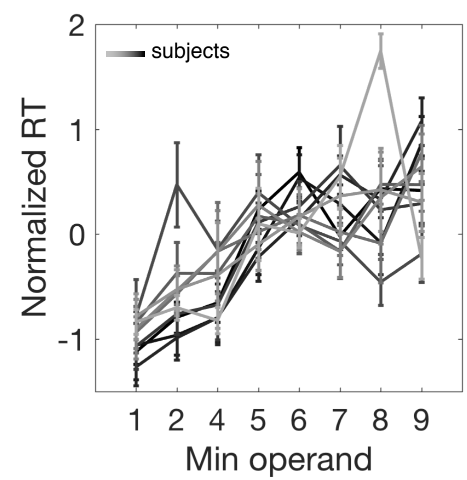
Self-paced
ITI 200 ms

Channel selectivity



All sites from all subjects projected into a single left hemisphere

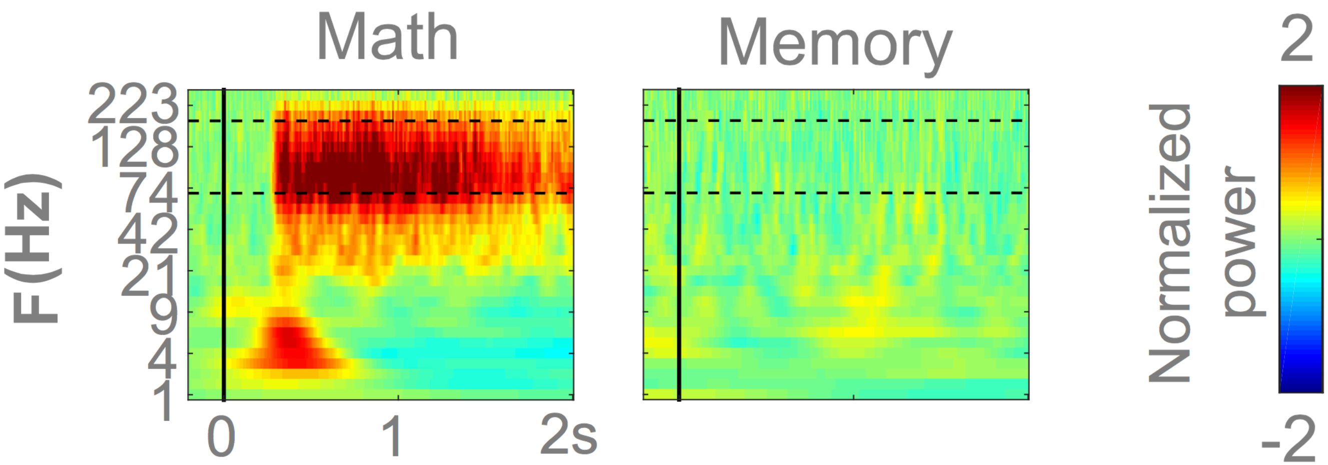
Problem-size effect



Best predictor of RT in 9/10 subjects

High frequency broadband (HFB): 70–180 Hz

high correlations with local spiking activity and the fMRI BOLD signal

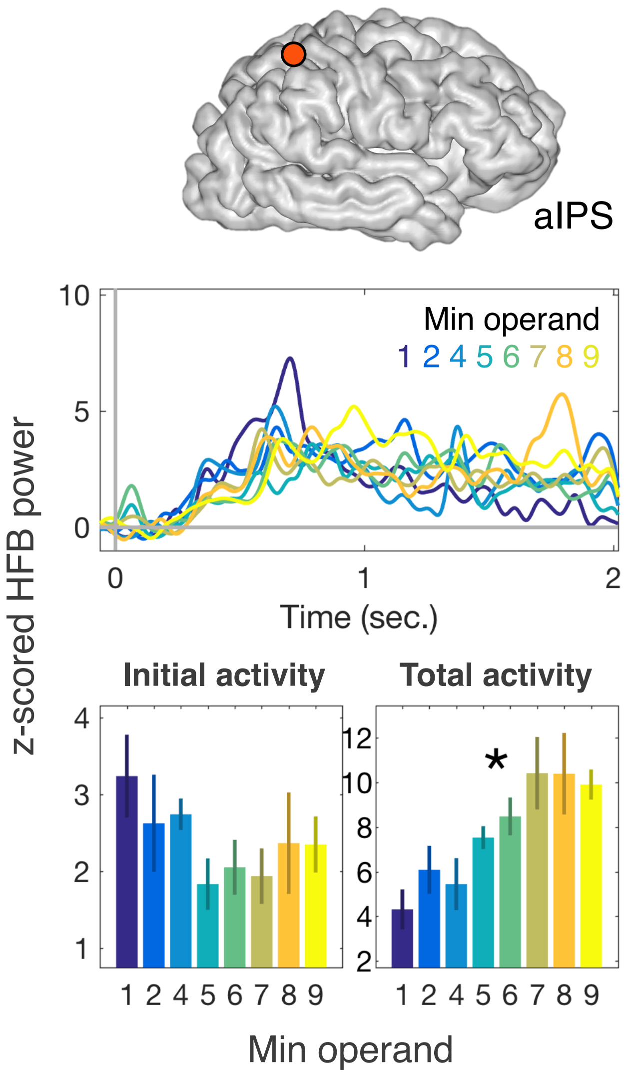


Exemplar aIPS channel

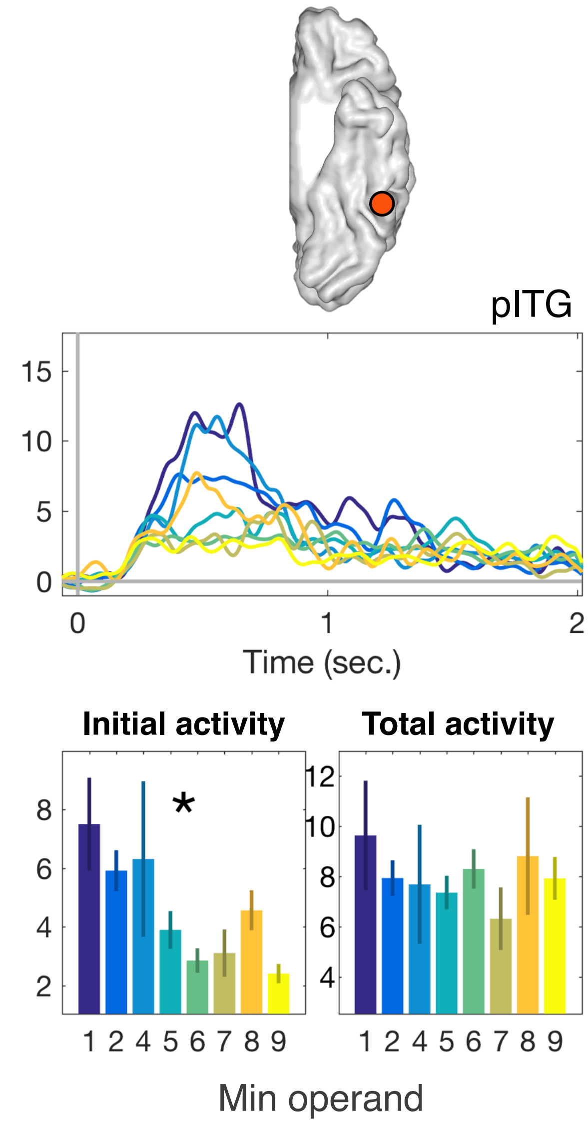
Two time windows:

- **initial activity : 1st second**
  averaged over the 1st second, when greatest increase in activity occurs.
- **total activity: integral**
  integral of the activity from trial onset to response: equivalent to BOLD.

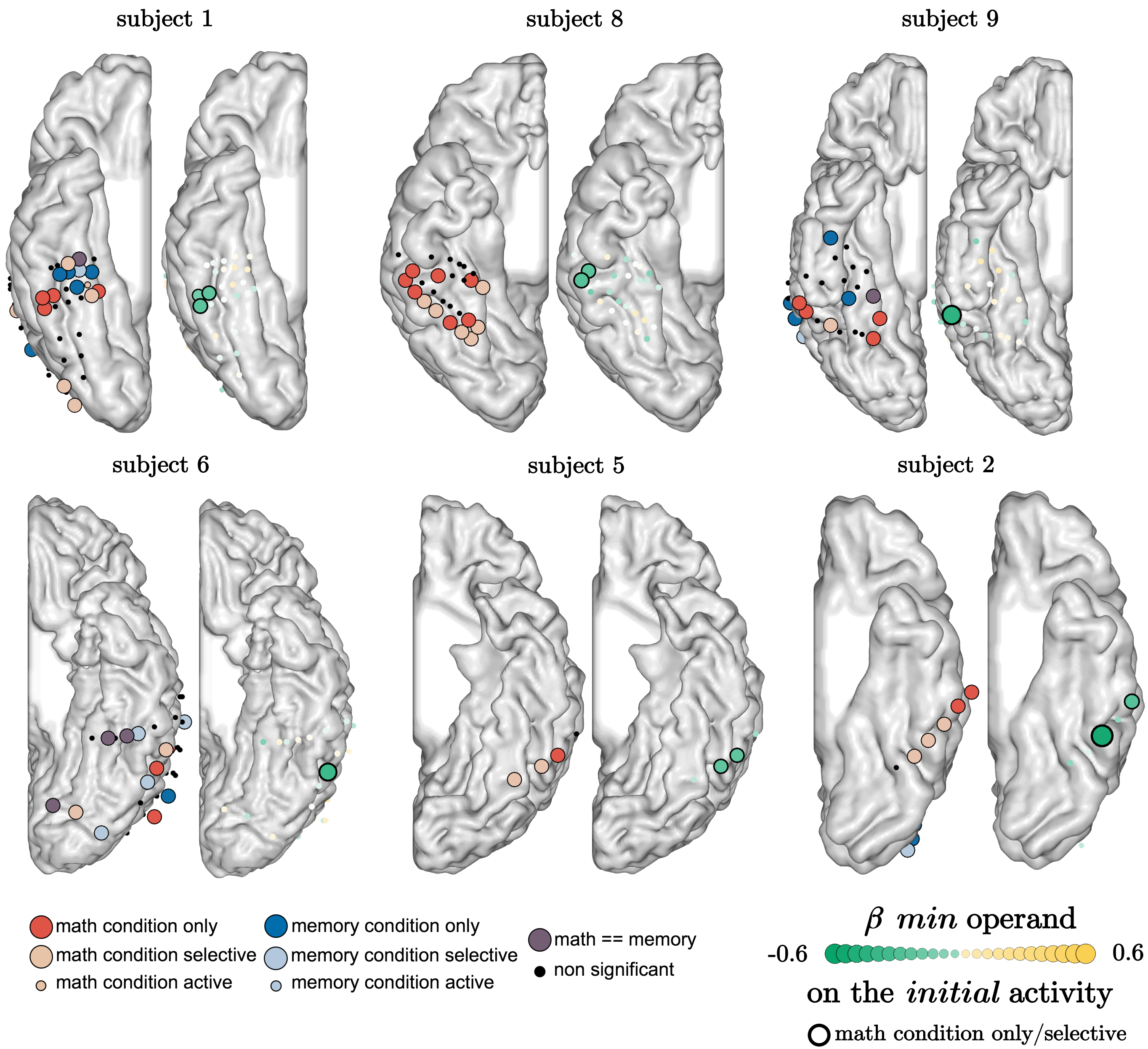
Increased ***total*** **activity by** ***min*** **operand in aIPS and SPL**



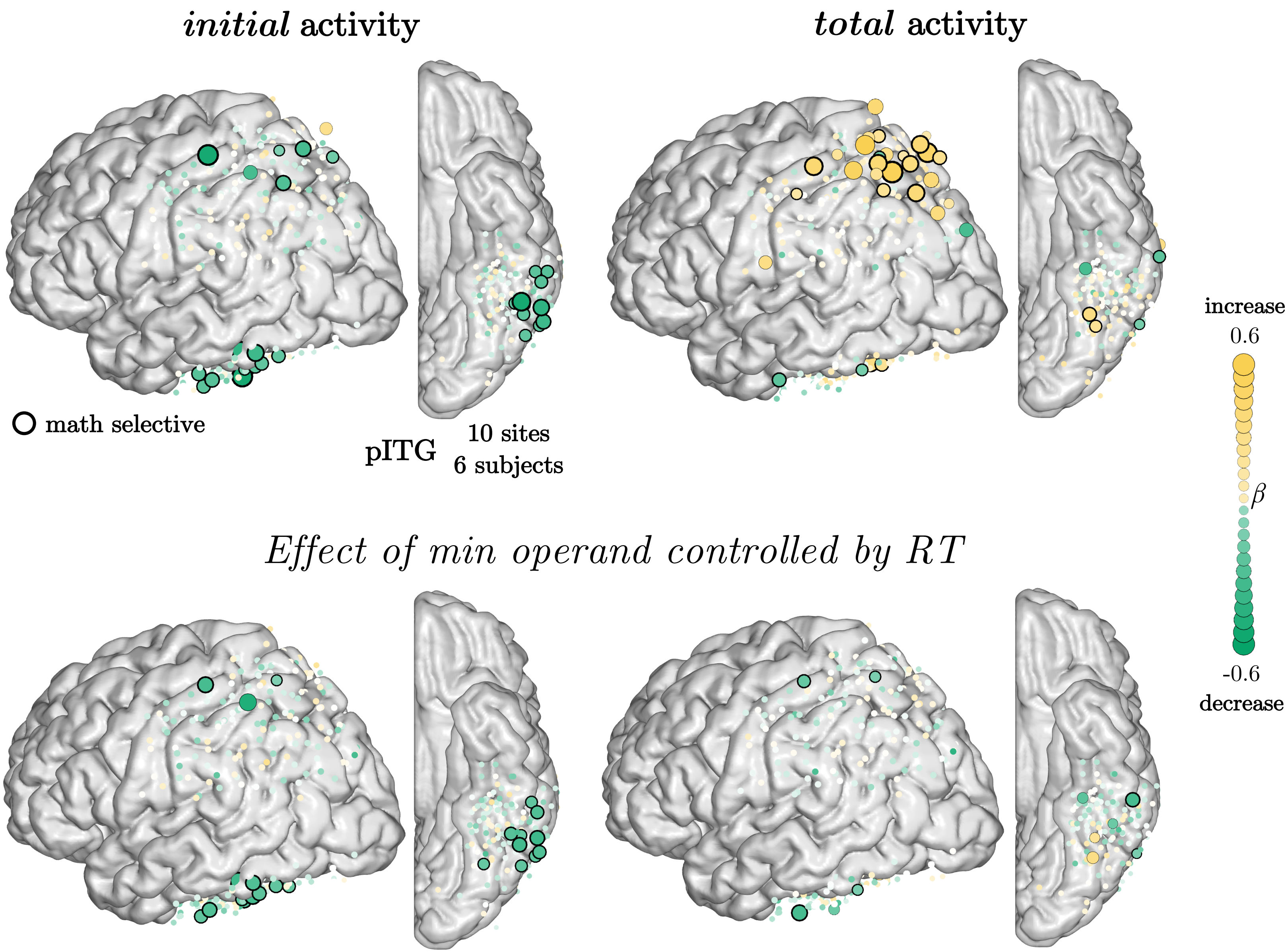
Decreased ***initial*** **activity by** ***min*** **operand in pITG**



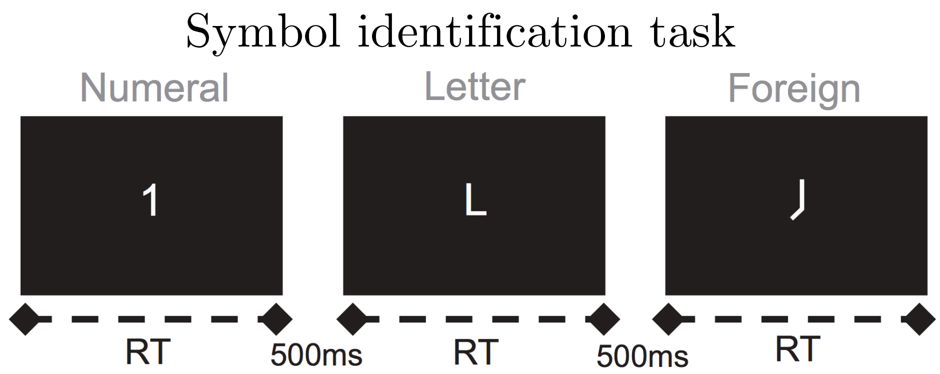
Anatomical and functional precision of the pITG modulation



Effect of the *min* operand on the:



Modulation of pITG does not reflect tuning to digits



Subject	Hemi	Region	Numeral active	Numeral selective	Modulation by numeral
S1	R	pITG	✓	-	-
S1	R	pITG	✓	-	-
S1	R	pITG	-	-	-
S2	L	pITG	✓	-	-
S2	L	aITG	-	-	-
S5	L	pITG	✓	-	-
S5	L	pITG	✓	-	-
S6	L	pITG	-	-	-
S8	R	pITG	✓	✓	-
S8	R	pITG	✓	-	-
S9	R	pITG	-	-	-

- **9/10 pITG** sites are **not NFA.**
- **pITG modulation** is exclusive to the **calculation** task.

Chapter 3. **Discussion and conclusions**

- **Modulation of aIPS and SPL corroborates previous fMRI findings.**
  Increased total activity as a function of problem-size.
  Constant initial activity; Slow and sustained – highly correlated with RT.
  Calculation and accumulation of evidence to achieve a decision.

- **Surprising role of pITG in mental arithmetic beyond digit recognition.**
  Decreased initial activity as a function of problem-size.
  Constant total activity: blind to fMRI.; Fast - not correlated with RT.
  Early identification of problem difficulty (amount of semantic evidence?).

- Re-evaluation of neurocognitive models of arithmetic and dyscalculia.
  pITG as an **important hub** for **calculation**.

- **Expands the classical view of the VTC**: contains regions specifically involved in sophisticated symbolic forms of reasoning, such as mental arithmetic.

Pinheiro-Chagas, Daitch*, Parvizi & Dehaene (under review)*

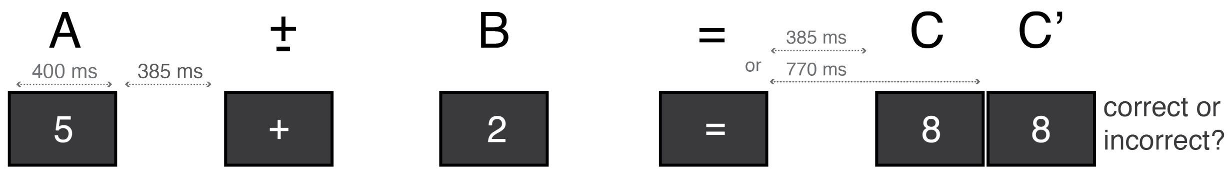
Chapter 4. **Decoding the processing stages of mental arithmetic with MEG**

- Lacking a comprehensive picture of the organization of the brain processes.
- Time-resolved multivariate pattern analysis applied to MEG.
- Can we decode the identity of the operands? Representational codes?
- Can we track in time the emergence of the internally computed result?
- Are the brain processes completely serial or do they partially overlap in a form of a cascade of computations that can be simultaneously decoded?

Pinheiro-Chagas, Piazza & Dehaene (under review)

Experimental Design

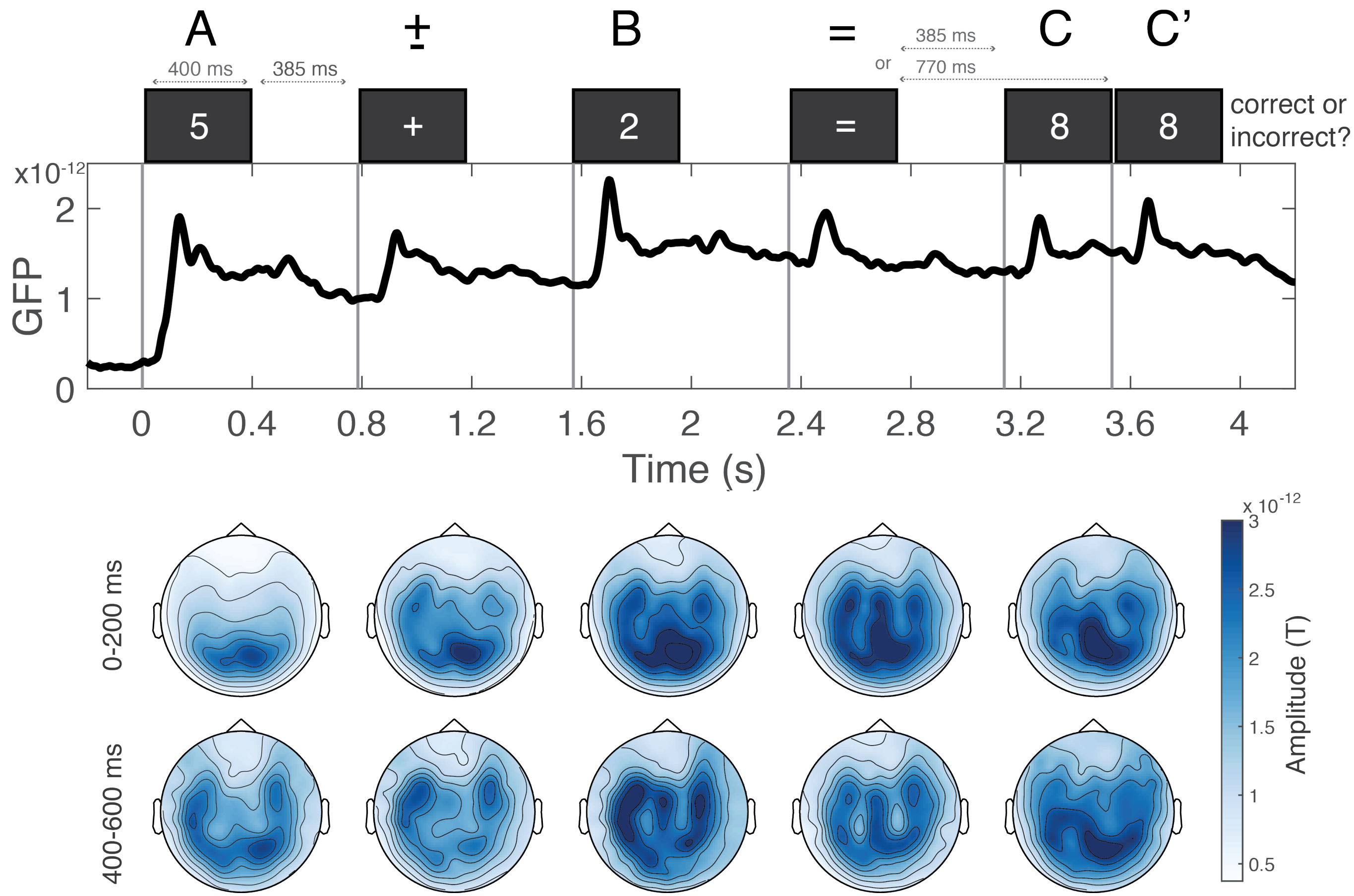
20 adults, students, right-handed



- **A** - *max* operand = **3, 4, 5, 6**
- **B** - *min* operand = **0, 1, 2, 3**
- **Result** = **0 – 9** (**3, 4, 5, 6** - 50 trials each)
- **C** – proposed result = **0 – 9**

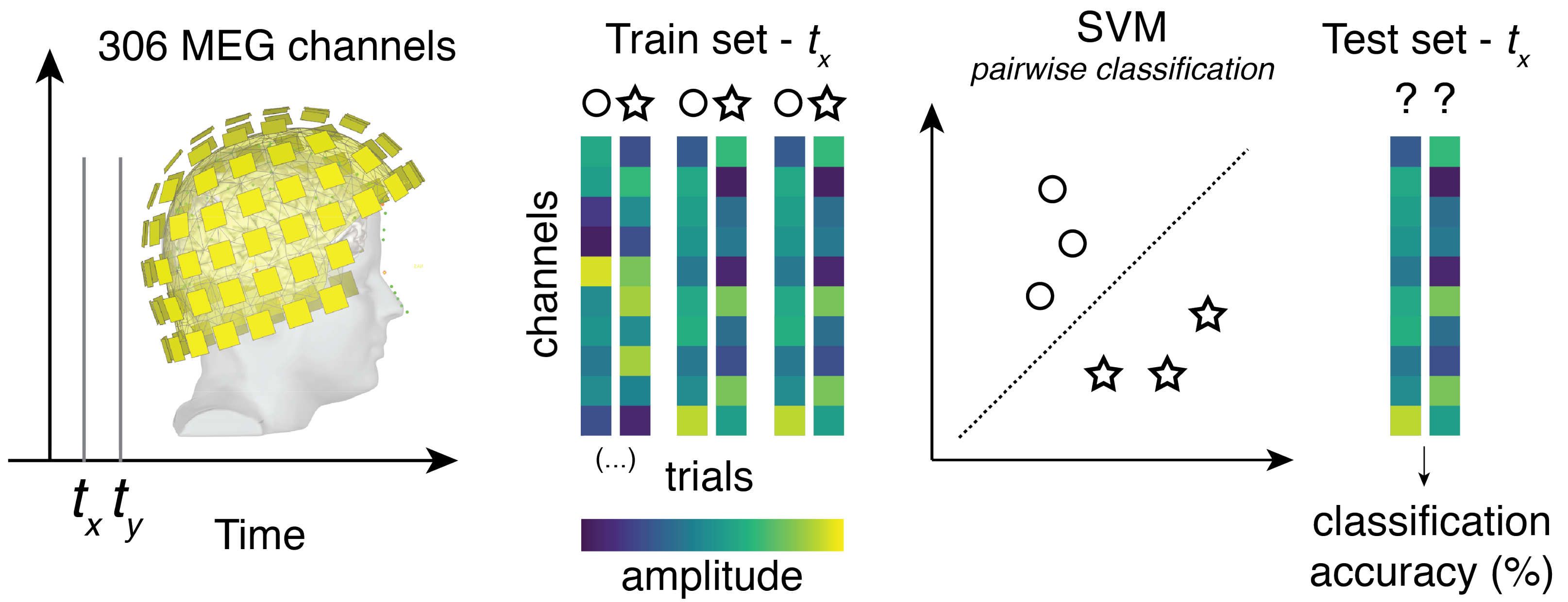
  50% incorrect (absolute distance: 1, 2, 3, 4)

Sustained activation from posterior to anterior sensors



- Apply time-resolved MPVA over the entire trial.

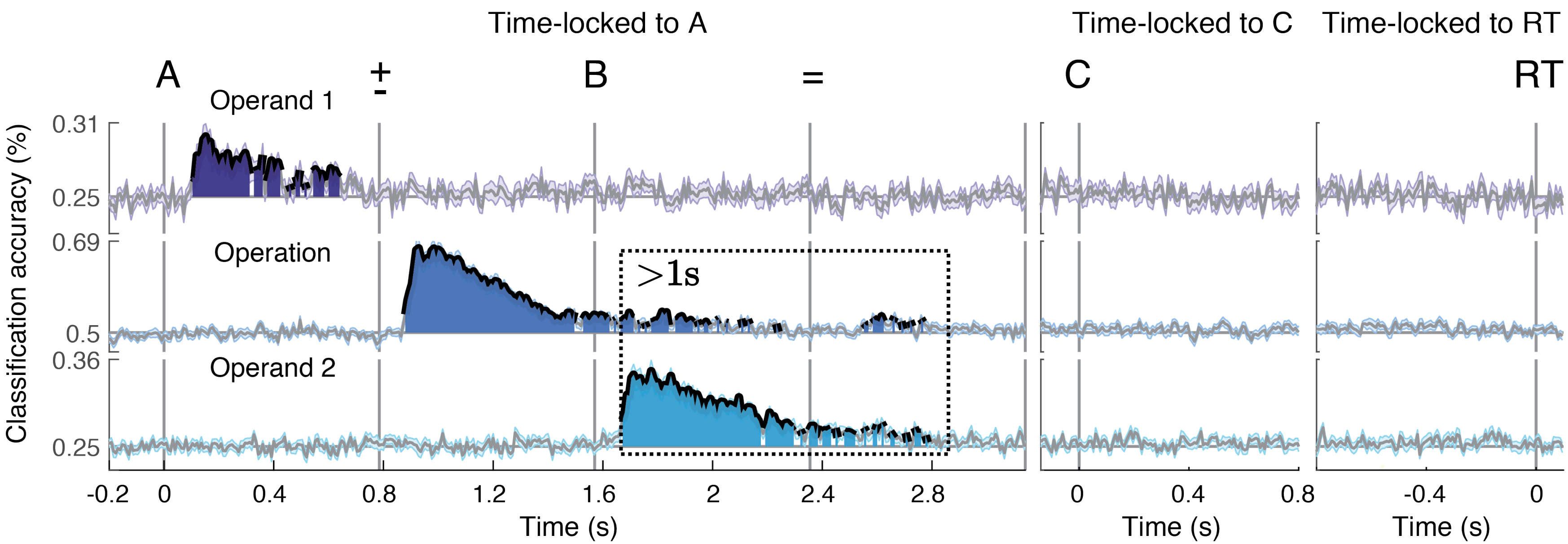
Time-resolved decoding



- The classifier is applied at each time sample t_x, t_y … , *per subject.*
- Generalization across time - can the classifier trained at t_x generalize to t_y?
  Test how stable in time are the underlying codes.
- Generalization across conditions.
  Test the existence of possible common codes.

(King & Dehaene, 2014)

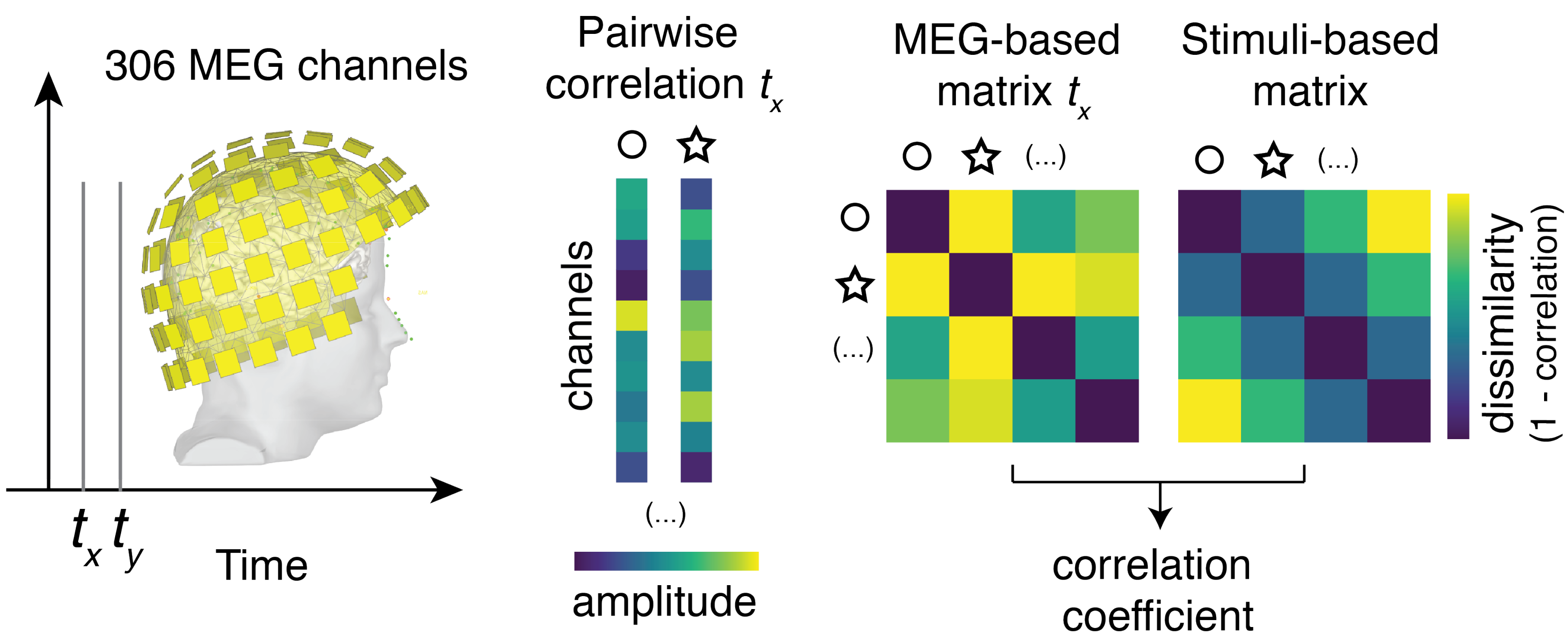
Decoding operands and operation



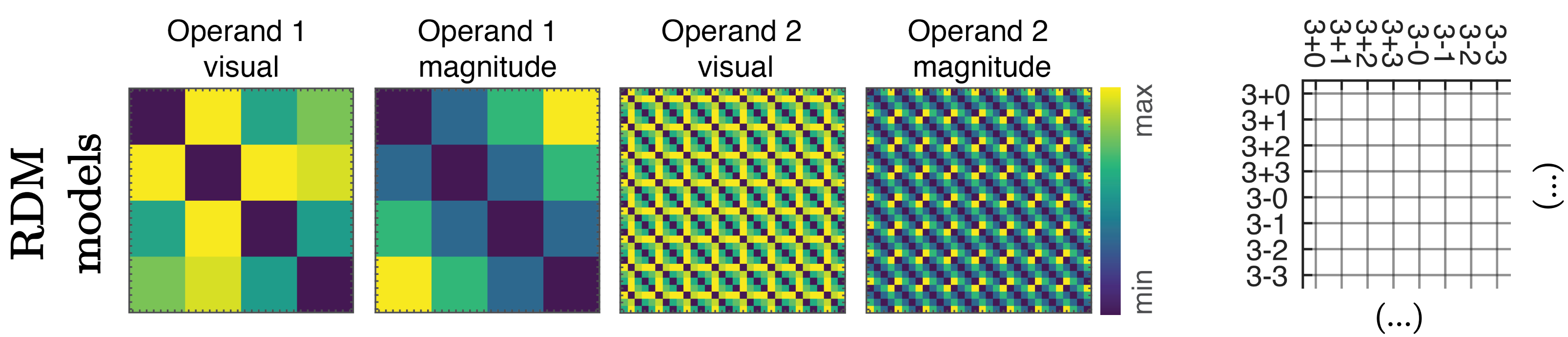
- **Operand 1**: **transient**, rapidly decreases after stim offset.
- **Operand 2: higher** accuracy and for a **longer** period – 1s.
- **Operation** *(sub vs. add)*: **sustained** for 2s, rebound after equal sign.
- Long **overlap** between **operation** and **operand 2** – 1s.

Temporal dynamics of the representational codes?

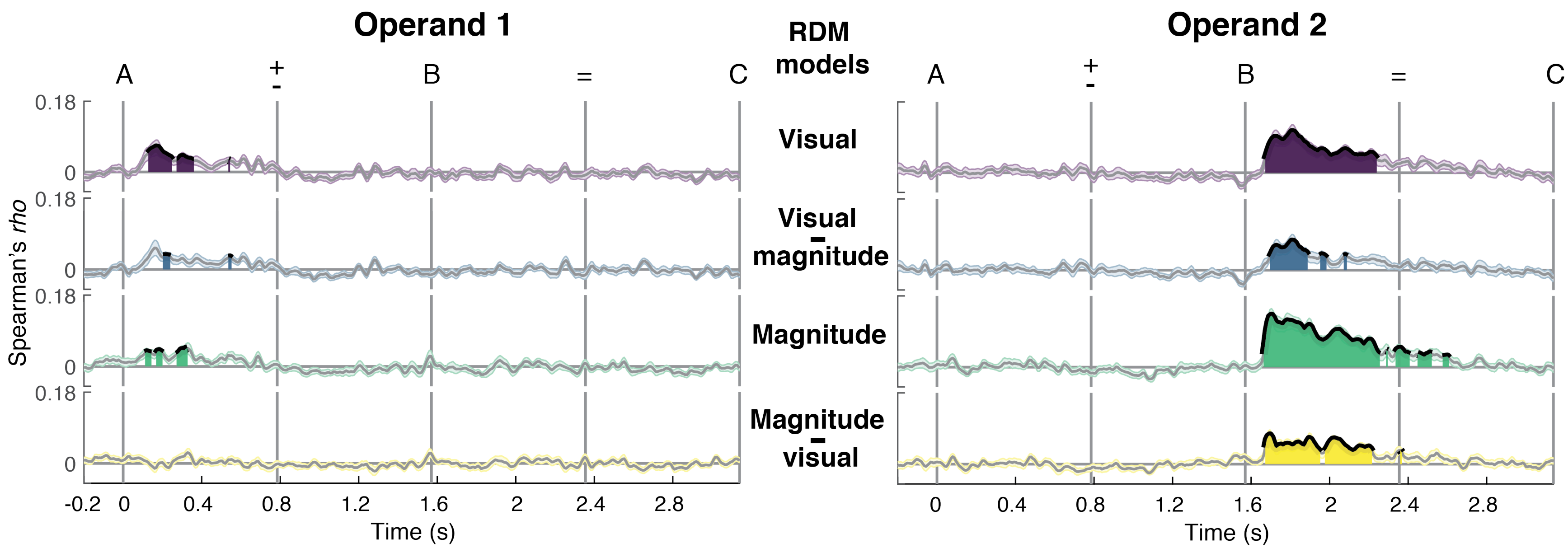
Time-resolved Representational Similarity Analysis



- The RSA is computed at each time sample $\boldsymbol{t_x}$, $\boldsymbol{t_y}$ ... , *per subject.*
- Simultaneously test the effect of different stimuli-based models.

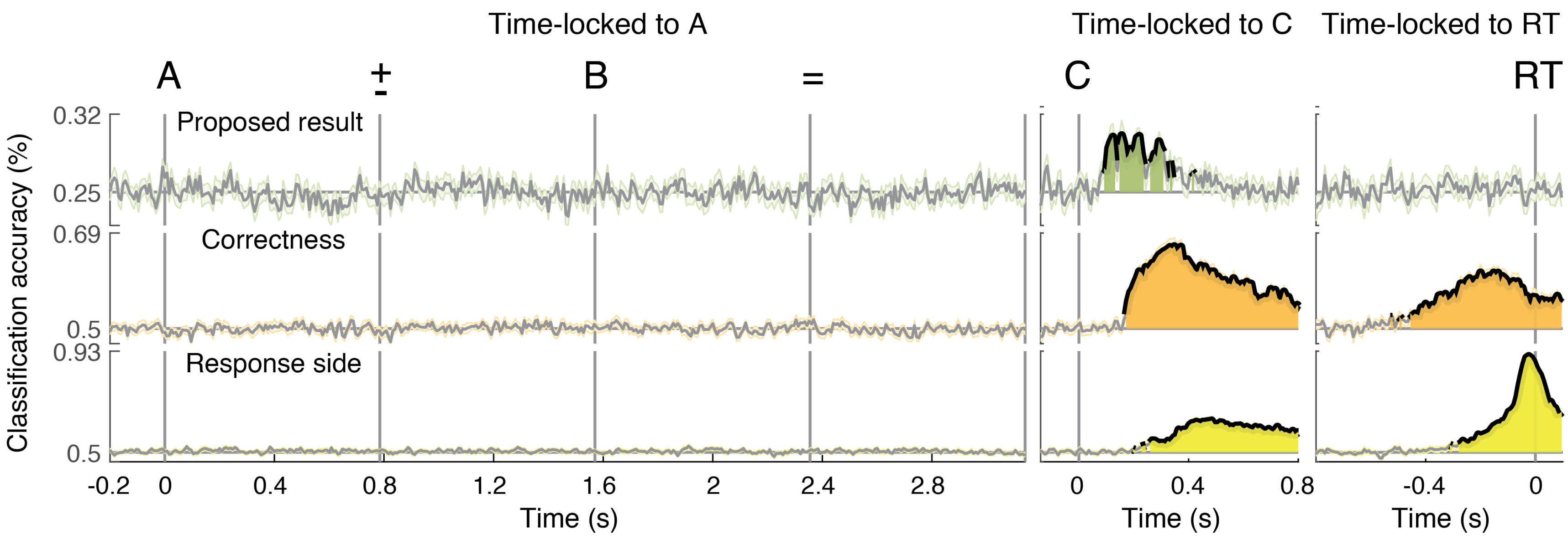


Representational geometries of the operands



- **Operand 1**: **visual dimension** is dominant.
- **Operand 2**: both visual and magnitude dimensions, **magnitude** dominant. *No precedence of the visual dimension.*

Highly overlapping dynamics at the decision stage

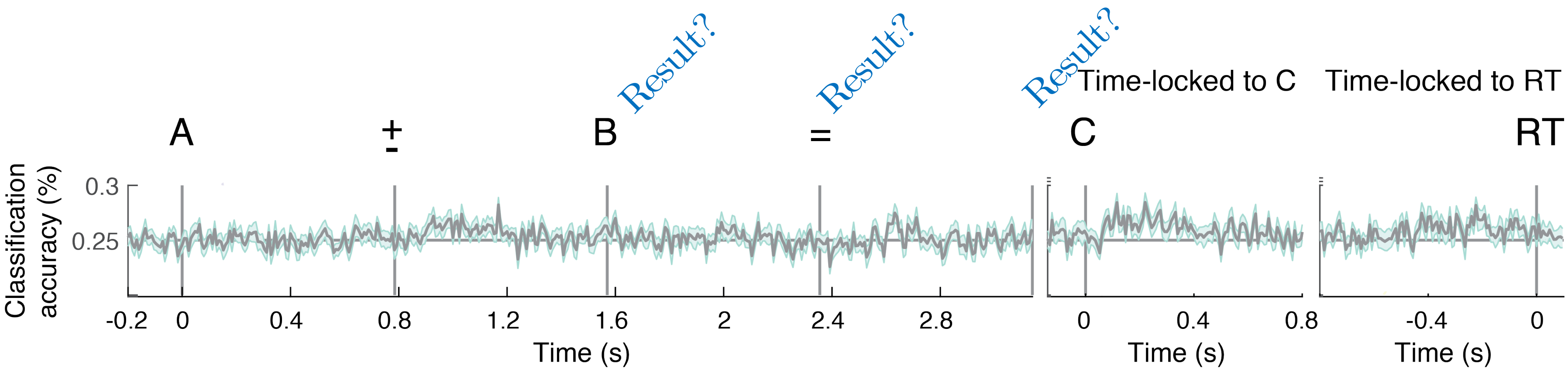


1. Identify the proposed result.
2. Judge whether it is correct or incorrect.
3. Press the response button.

Searching for a neural signature of the internally computed result

Several attempts, no success

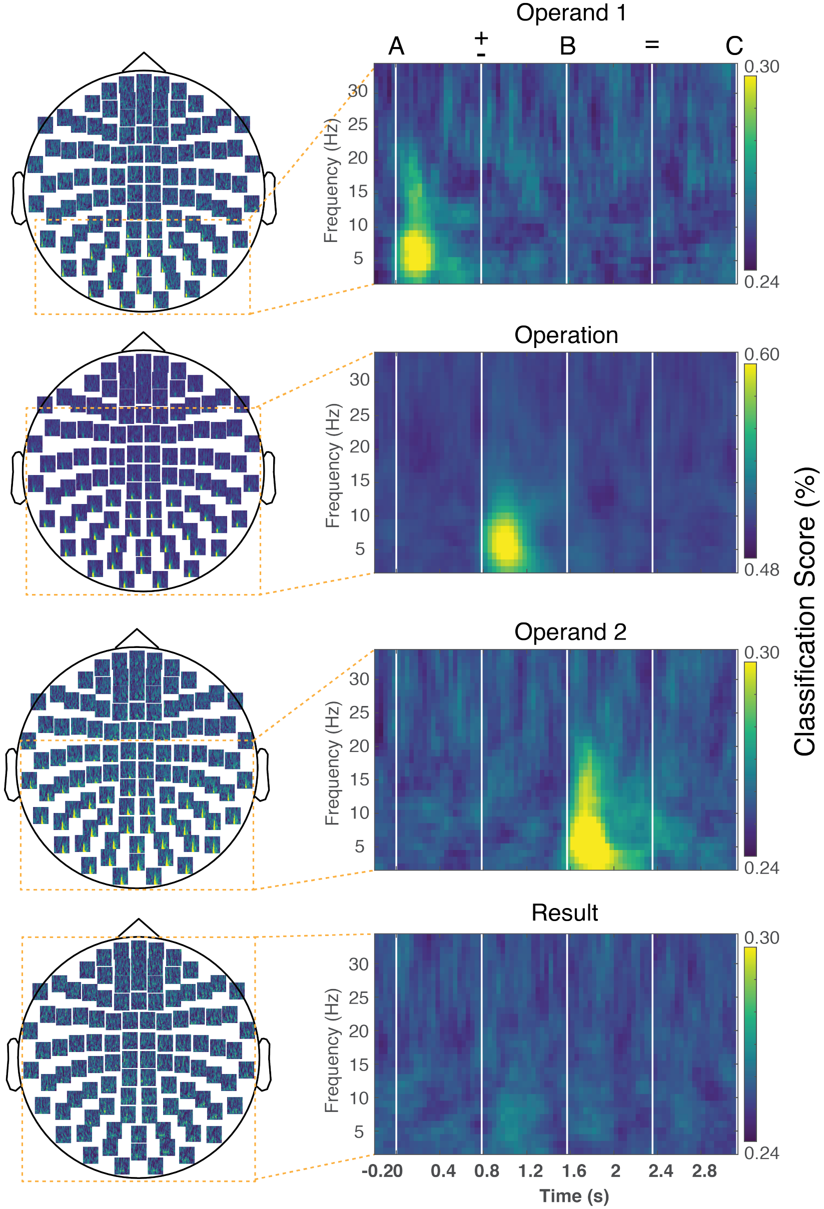
Time-resolved decoding



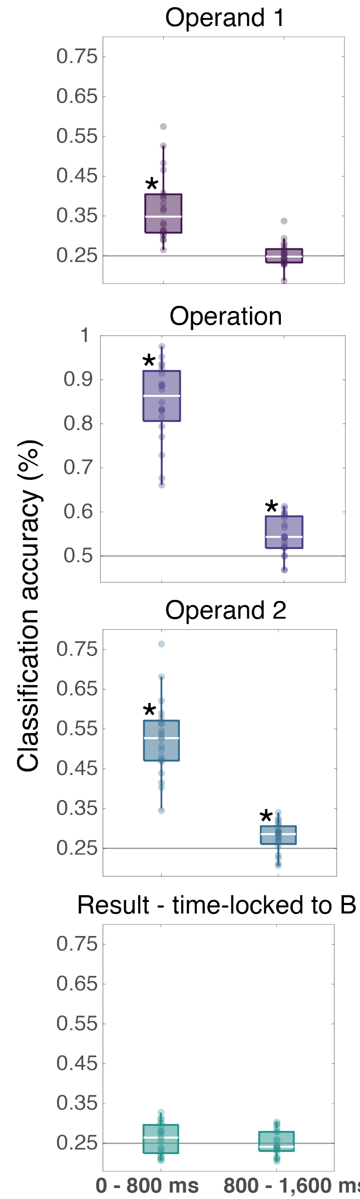
Searching for a neural signature of the internally computed result

Several attempts, no success

Searchlight LDA (sensor, time and frequency)



Riemannian Geometry (integrated over time)



Chapter 4. **Discussion and conclusions**

- First **comprehensive picture** of the **unfolding processing stages** underlying **arithmetic** calculations at a single-trial level.

- Decoding of **operand 1**: low, transient - **visual dimension.**

- Decoding **operand 2 *(min)***: high, sustained - visual and **magnitude** dimensions;
  Additional neuronal populations recruited in the VTC and LPC?

- **Fast** and **highly overlapping dynamics** at the **decision stage:**
  1. Identify the proposed result.
  2. Judge whether it is correct or incorrect.
  3. Press the response button.

- **Inability to decode the internally computed result**
  - Externally and internally generated codes have different neural substrates.
  - Internally generated: rare gamma bursts, sparse code, *'silent states'*?
    Hard to capture with MEG.
  - Limitations of the design (n_{trials}, no time-stamping, etc.).

Pinheiro-Chagas, Piazza & Dehaene (under review)

General conclusions and future directions

General conclusions and future directions

- **Single-digit addition and subtraction rely on quantity manipulation:**
  - **Operands** are processed **serially**: larger ➤ smaller.
  - **Stepwise displacement** on the **mental number line**.
  - *Precise mechanisms of the serial processing and learning?*

- **Arithmetic is implemented in the dorsal and ventral pathways:**
  - IPS and SPL: arithmetic computations and decision-making.
  - **pITG**: beyond digit recognition: **early identification** of **problem difficulty**.
  - *How do they interact? IPS integrates the activity of pITG? PFC?*

- **Decoding the processing stages of mental calculation:**
  - Cascade of highly dynamic and partially overlapping brain states.
  - Operand 1: visual; **Operand 2** (***min***): visual and **magnitude** codes.
  - *Temporal evolution from symbol to quantity?*

- **Searching for neural signatures of the internally computed result:**
  - Inability to decode with MEG.
  - *When and where is it generated?*

- **Beyond numbers: the syntactic structure of arithmetic expressions**

Stay tuned for the next articles

Pinheiro-Chagas, P., Dotan, D., Piazza, M., Dehaene, S. (*in preparation*). Decomposing the syntactic structure of arithmetic expressions.

Dotan, D., **Pinheiro-Chagas, P.**, Dehaene, S. (*in preparation*) Track it to crack it: revealing the succession of processing stages with pointing trajectories.

Baek, S., Daitch, A., **Pinheiro-Chagas, P**., Parvizi, J. (*under revision*). Neuronal population responses in the human ventral temporal and lateral parietal cortex during arithmetic processing with digits and number words.

Other articles produced during the dissertation

Dresler, T., Bugden, S., Gouet, C., Lallier, M., Oliveira, D., **Pinheiro-Chagas, P.**, Pires, A., Wang, Y., Zugarramurdi, C., Weissheimer, J. (*under review*). Translational research in learning disabilities: the place of neuroimaging.

Borghesani*, V., de Hevia*, L., Viarouge*, A., **Pinheiro-Chagas, P.**, Eger, E., Piazza, M. (*under review*). Processing number and length in the parietal cortex: sharing resources, not a common code.

Pinheiro-Chagas, P.*, Dinino, D.*, Haase, V. G., Wood, G., Knops, A. (*in preparation*) The developmental trajectory of the operational momentum effect.

Borghesani*, V., de Hevia*, L., Viarouge*, A., **Pinheiro-Chagas, P.**, Eger, E., Piazza, M. (2016). Comparing magnitudes across dimensions: a univariate and multivariate approach. International Workshop on Pattern Recognition in Neuroimaging, 1-4.

Pinheiro-Chagas, P. Wood, G., Knops, A., Krinzinger, H., Lonnemann, J., Starling-Alves, I., Willmes, K., Haase, V. G. (2014). In how many ways is the approximate number system associated with exact calculation? *PLoS One*, 19, 9(11), e111155.

Carvalho, M. R., Vianna, G., Oliveira, L., Costa, A. J., **Pinheiro-Chagas, P.**, Sturzenecker, R., Zen, P. R., Rosa, R. F., de Aguiar, M. J., Haase, V. G. (2014). Are 22q11.2 distal deletions associated with math difficulties? *American Journal of Medical Genetics Part A*, 164A(9), 2256-62.

Haase, V. G., Júlio-Costa, A., Lopes-Silva, J. B., Starling-Alves, I., Antunes, A. M., **Pinheiro-Chagas, P.**, Wood, G. (2014). Contributions from specific and general factors to unique deficits: two cases of mathematics learning difficulties. *Frontiers in Psychology*, 13, 5-102.

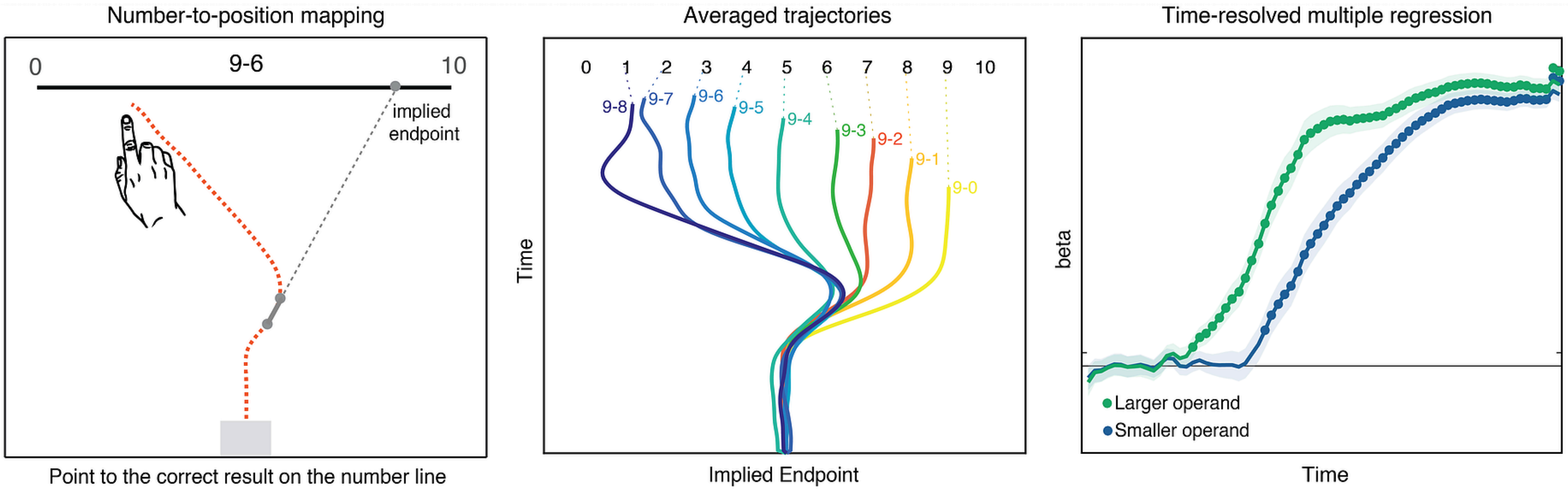
Moura, R., Wood, G., **Pinheiro-Chagas, P.**, Lonnemann, J., Krinzinger, H., Willmes, K., Haase, V. G. (2013). Transcoding abilities in typical and atypical mathematics achievers: the role of working memory and procedural and lexical competencies. *Journal of Experimental Child Psychology*, 116(3), 707-27

* The authors equally contributed to the work

TrajTracker: track it to crack it

Platform for trajectory tracking experiments & data analysis

TrajTracker Experiment 1.0 (beta version) is now released!
Android support will be available with the next release of Expyriment for Android.



tracking mental calculation online

TrajTracker is a software package for running and analyzing psychology experiments, focused on mouse/finger tracking experiments.

New to trajectory tracking? Read about it here.



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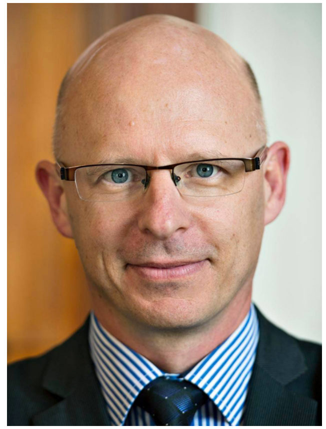
Volunteers



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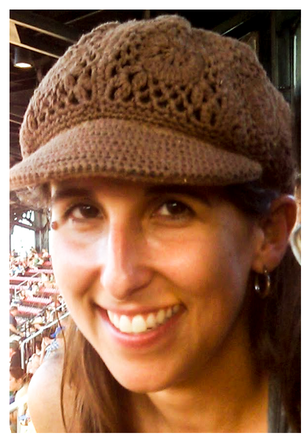

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